The radial stress (pressure) in terms of displacement [14] is

$$\sigma = (\lambda + 2\mu)(\partial u/\partial r) + (2\lambda/r)u - \beta v \quad (31)$$

and so we have, by substituting (29) into (31),

$$\sigma = u_0 G + \sum_{m=1}^{\infty} A_m k_m H_m \cos \omega_m t \quad (32)$$

where

$$G = -(4\mu a/N\pi r)j_1(N\pi r/a) \mp (1/N\pi)^2(3\lambda + 2\mu), \quad N = \begin{cases} 1, 3, 5, \cdots \\ 2, 4, 6, \cdots \end{cases} \quad (33)$$

$$H_m = (\lambda + 2\mu)j_0(k_m r) - (4\mu/k_m r)j_1(k_m r). \quad (34)$$

Solution for a Rectangular Pulse

We are now in the position to apply the above results to obtain the appropriate expressions for the displacement and pressure due to a short rectangular pulse of microwave energy with pulsewidth t_0. As previously mentioned, Duhamel's principle [23] allows us to find the solution u for any $F_t(t)$ once the solution u' for $F_t(t) = 1$ is known. The method is to apply the formula

$$u = (\partial/\partial t) \int_0^t F_t(t - t')u'(r,t')\, dt'. \quad (35)$$

Therefore, by substituting (10) and (29) into (35), we have for the radial displacement

$$u = u_0 Dt + \sum_{m=1}^{\infty} A_m j_1(k_m r)(\sin \omega_m t/\omega_m), \qquad 0 < t < t_0 \quad (36)$$

$$u = u_0 Dt_0 + \sum_{m=1}^{\infty} A_m j_1(k_m r) \cdot [\sin \omega_m t/\omega_m - \sin \omega_m(t - t_0)/\omega_m], \qquad t > t_0. \quad (37)$$

Similarly, we have for the pressure

$$\sigma = u_0 Gt + \sum_{m=1}^{\infty} A_m k_m H_m(\sin \omega_m t/\omega_m), \qquad 0 < t < t_0 \quad (38)$$

$$\sigma = u_0 Gt_0 + \sum_{m=1}^{\infty} A_m k_m H_m[\sin \omega_m t/\omega_m - \sin \omega_m(t - t_0)/\omega_m], \qquad t > t_0 \quad (39)$$

where D, G, and H_m are as given in (30), (33), and (34). Equations (36)–(39) represent the general solution for the radial displacement and pressure in a spherical head model with the constrained boundary exposed to a rectangular pulse of microwave radiation. It is seen that the displacement becomes zero both at the center and at the surface of the spherical head.

Since u_0 and A_m are proportional to I_0, both the displacement and the pressure are proportional to the peak absorption, similar to the stress-free boundary case. The dependence on total pulse energy $I_0 t_0$, however, is not as clear cut.

TABLE II
ZEROS OF THE SPHERICAL BESSEL FUNCTION $j_1(ka) = 0$

m	$k_m a$
1	4.493411
2	7.725252
3	10.904122
4	14.066194
5	17.220755
6	20.371303
7	23.519452
8	26.666054
9	29.811599
10	32.956389
11	36.100622

NUMERICAL RESULTS

We may use the results of the last section to estimate the frequency and amplitude of acoustic signals generated in the heads of animals and humans exposed to rectangular pulses of microwave energy. The useful physical parameters of brain matter are listed in Table I.

Frequency of Sound

The frequency of vibration of the spherical head is derived from (25). As mentioned before, there are an infinite number of resonant frequencies; each corresponding to a mode of vibration of the spherical head. The first 11 zeros of $j_1(k_m a)$ are given in Table II. It is seen that the frequency of vibration is completely independent of the microwave absorption pattern; it is only a function of the size of the spherical head and the acoustic properties of the tissue involved. This indicates that the frequency of sound perceived by a subject irradiated by rectangular pulses of microwave energy will be the same regardless of the frequency of the impinging radiation.

The fundamental frequency as seen from (27) and Table II is

$$f_1 = k_1 c_1/2\pi = 4.49c_1/(2\pi a). \quad (40)$$

The fundamental frequency is plotted as a function of spherical head radius in Fig. 2. It is readily observed that the frequency in various subjects differs according to their equivalent spherical head sizes, i.e., the smaller the head size, the higher the frequency. For example, the average head radius for guinea pigs is about 1.5–2.5 cm; Fig. 2 yields a range of 40–70 kHz for the corresponding fundamental sound frequency. The average head radius for cats is approximately 2.5–3.5 cm; the corresponding fundamental sound frequency is between 30 and 40 kHz. It is significant to note that these frequencies are very close to the 50-kHz cochlear microphonics reported for guinea pigs [7] and the 38-kHz oscillations reported for cats [24]. Human head sizes are known to vary from 7 to 10 cm for adults. From Fig. 2, we see that the estimated fundamental sound frequency ranges from 10 to 15 kHz. This is certainly not in violation of the